

Difference-in-Differences

PP5001 · Week 8 · Martinmas 2026

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The Evaluation Problem

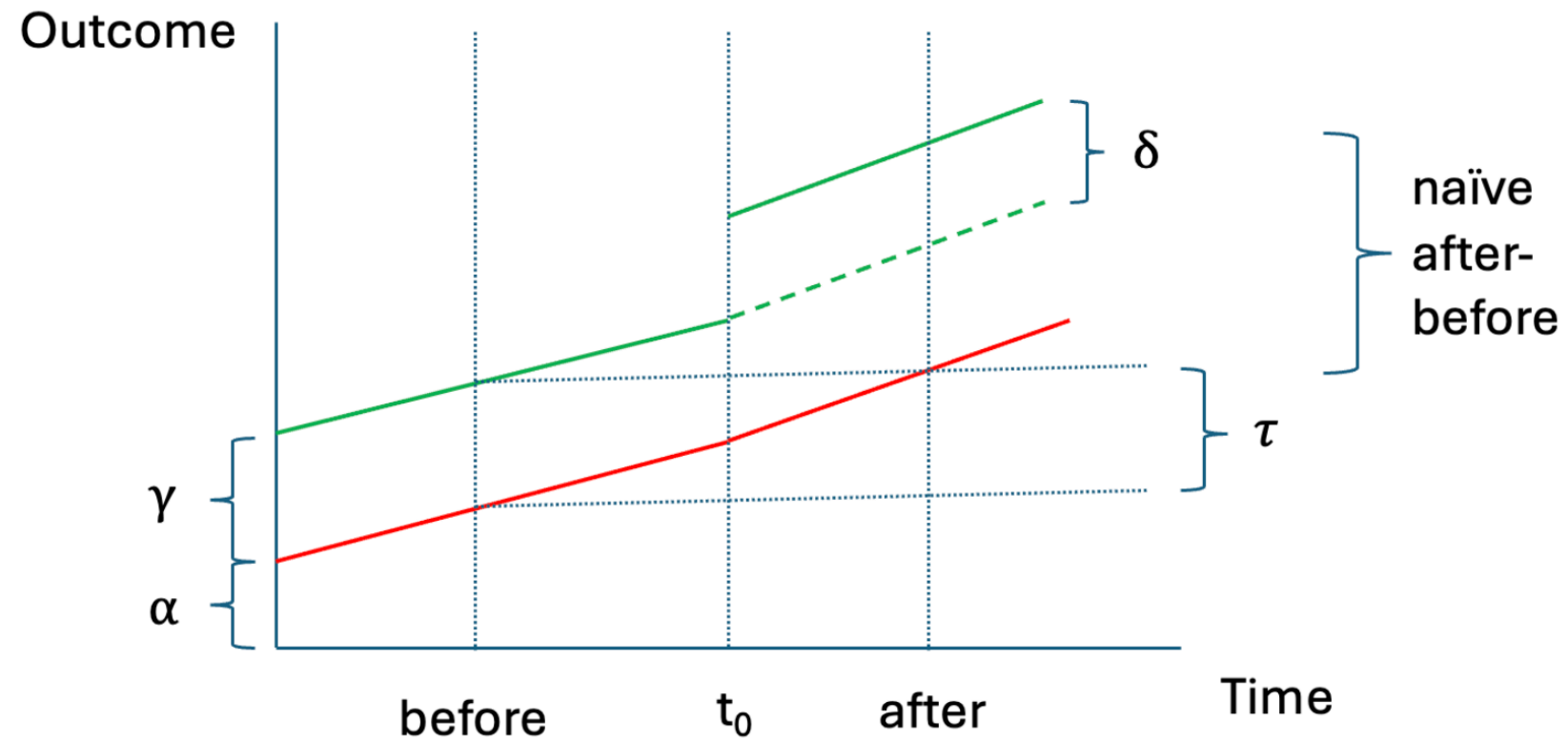
A policy begins at a particular time in a particular place. We observe outcomes before and after its introduction.

The before–after change combines:

- The effect of the policy.
- The change that would have occurred even without the policy.

We need a comparison that tells us about that second component.

Before and After



The Basic DiD Idea

Use the change in the control group to estimate the change the treated group would have experienced without treatment.

$$\hat{\delta}_{DiD} = (\bar{Y}_{T,post} - \bar{Y}_{T,pre}) - (\bar{Y}_{C,post} - \bar{Y}_{C,pre}) .$$

Taking each group's change removes its fixed level. Subtracting the control group's change removes the common change over time.

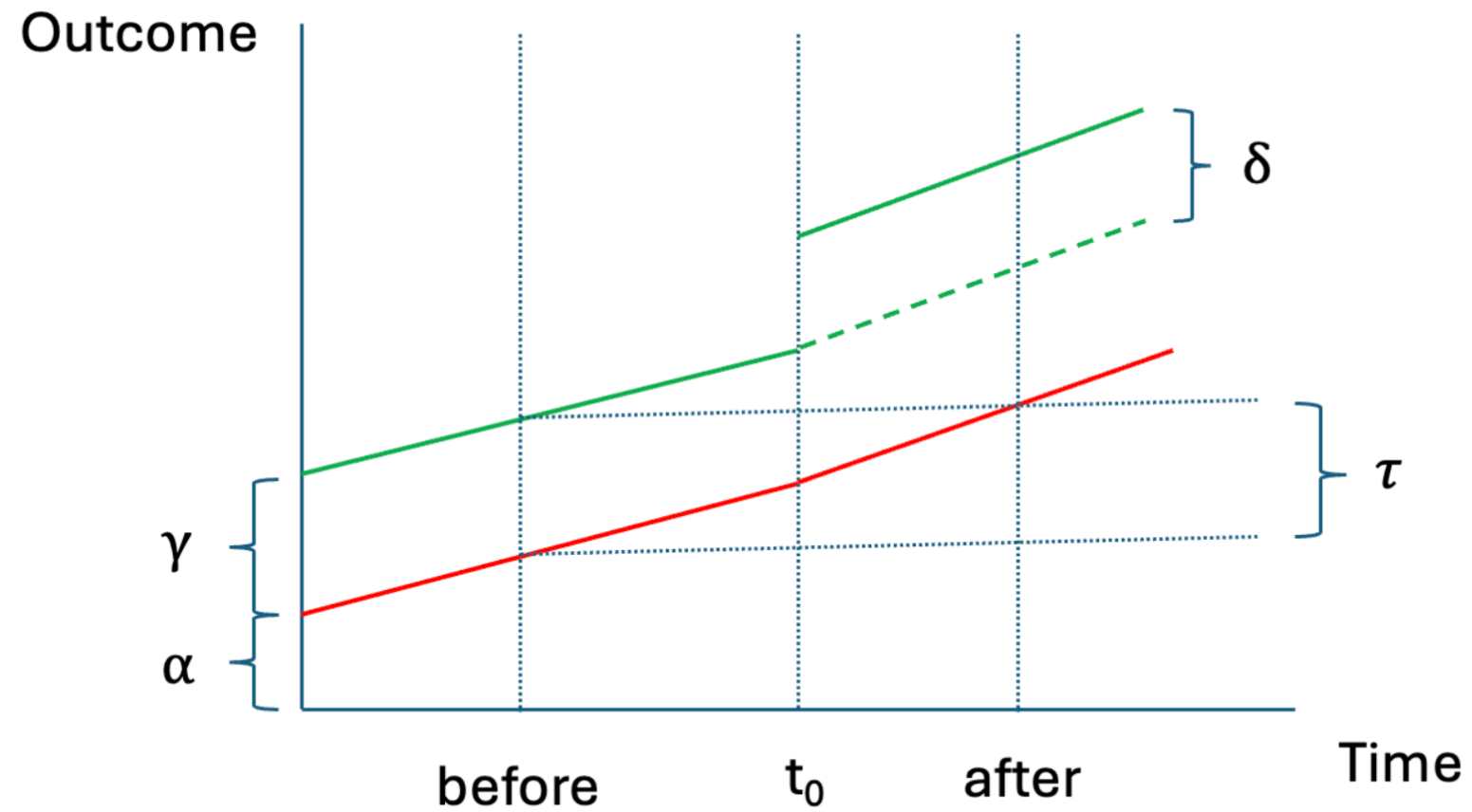
The 2×2 Parameterization

Group	Before	After	Change
Treatment	$\alpha + \gamma$	$\alpha + \gamma + \tau + \delta$	$\tau + \delta$
Control	α	$\alpha + \tau$	τ
Treatment minus control	γ	$\gamma + \delta$	δ

α : control-group baseline. γ : initial group difference.

τ : common time change. δ : the additional change in the treated group.

The Parameters in the Graph



Regression Representation

Let $T_i = 1$ identify membership in the treatment group and $Post_t = 1$ identify the post-policy period.

$$Y_{it} = \alpha + \gamma T_i + \tau Post_t + \delta (T_i \times Post_t) + u_{it}.$$

- T_i allows the groups to have different initial levels.
- $Post_t$ allows for the common time change.
- $T_i \times Post_t$ equals one only for treated observations after the policy.

Why the Interaction Coefficient Is DiD

With an intercept and these three indicators, OLS fits all four cell means:

$$\hat{\alpha} = \bar{Y}_{C,pre},$$

$$\hat{\gamma} = \bar{Y}_{T,pre} - \bar{Y}_{C,pre},$$

$$\hat{\tau} = \bar{Y}_{C,post} - \bar{Y}_{C,pre},$$

$$\hat{\delta} = \bar{Y}_{T,post} - \hat{\alpha} - \hat{\gamma} - \hat{\tau}.$$

Substituting the first three expressions into the last gives the difference-in-differences estimator.

Potential Outcomes and the ATT

Let Y_{it}^1 and Y_{it}^0 denote outcomes with and without treatment.

The effect of interest is

$$ATT = E \left[Y_{i,post}^1 - Y_{i,post}^0 \mid T_i = 1 \right] .$$

We observe $Y_{i,post}^1$ for the treatment group. Its mean untreated outcome after the policy is the missing counterfactual.

Assume no anticipation and no spillovers to the control group.

Parallel Trends

In the absence of treatment, the two groups would have experienced the same average change:

$$E [Y_{i,post}^0 - Y_{i,pre}^0 \mid T_i = 1] = E [Y_{i,post}^0 - Y_{i,pre}^0 \mid T_i = 0] .$$

The groups may start at different levels. The assumption concerns their **changes in untreated potential outcomes**.

Constructing the Counterfactual

Parallel trends implies

$$E [Y_{i,post}^0 \mid T_i = 1] = E [Y_{i,pre}^0 \mid T_i = 1] \\ + E [Y_{i,post}^0 - Y_{i,pre}^0 \mid T_i = 0] .$$

Treated group's baseline + control group's change = treated group's counterfactual post-policy mean.

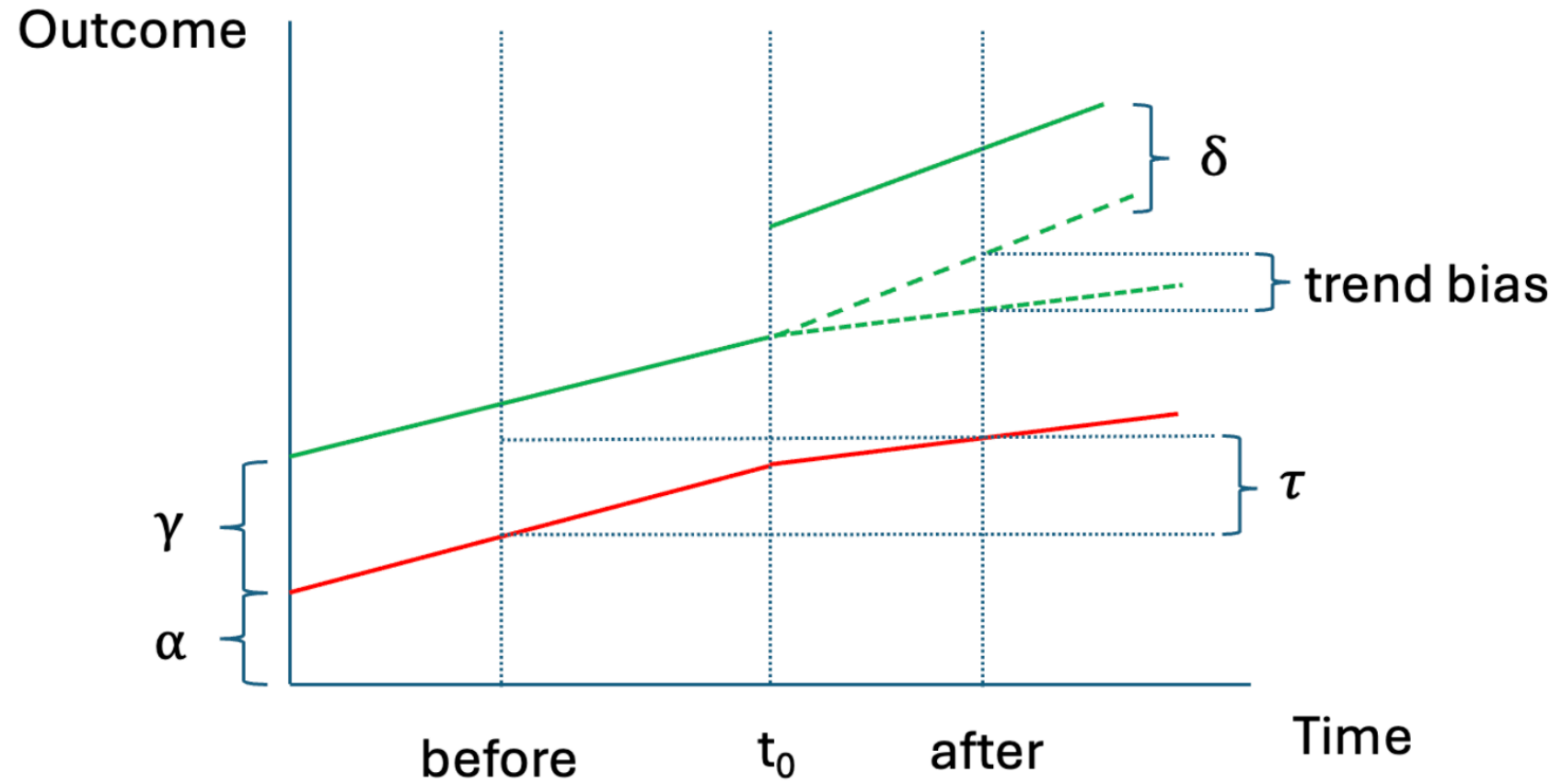
From the Counterfactual to the Effect

Substitute that counterfactual into the definition of the ATT:

$$\begin{aligned} ATT = E[Y_{i,post} \mid T_i = 1] - E[Y_{i,pre} \mid T_i = 1] \\ - (E[Y_{i,post} \mid T_i = 0] - E[Y_{i,pre} \mid T_i = 0]) . \end{aligned}$$

Use the analogy principle to replace the four population conditional means with their sample counterparts.

Different Underlying Trends



Card and Krueger: Policy, Data, and Identification

For discussion: What changed in New Jersey? Who was surveyed, when, and why is eastern Pennsylvania a plausible control group? State the identifying assumption in this setting.

Card and Krueger: Figure 1

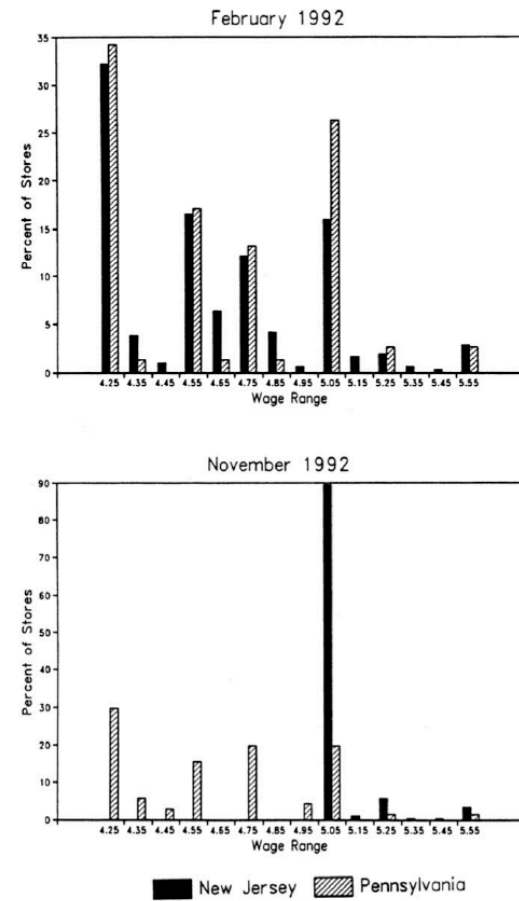


FIGURE 1. DISTRIBUTION OF STARTING WAGE RATES

Card and Krueger (1994), Figure 1.

Card and Krueger: Table 3

TABLE 3—AVERAGE EMPLOYMENT PER STORE BEFORE AND AFTER THE RISE
IN NEW JERSEY MINIMUM WAGE

Variable	Stores by state			Stores in New Jersey ^a			Differences within NJ ^b	
	PA (i)	NJ (ii)	Difference, NJ – PA (iii)	Wage = \$4.25 (iv)	Wage = \$4.26–\$4.99 (v)	Wage ≥ \$5.00 (vi)	Low– high (vii)	Midrange– high (viii)
1. FTE employment before, all available observations	23.33 (1.35)	20.44 (0.51)	– 2.89 (1.44)	19.56 (0.77)	20.08 (0.84)	22.25 (1.14)	– 2.69 (1.37)	– 2.17 (1.41)
2. FTE employment after, all available observations	21.17 (0.94)	21.03 (0.52)	– 0.14 (1.07)	20.88 (1.01)	20.96 (0.76)	20.21 (1.03)	0.67 (1.44)	0.75 (1.27)
3. Change in mean FTE employment	– 2.16 (1.25)	0.59 (0.54)	2.76 (1.36)	1.32 (0.95)	0.87 (0.84)	– 2.04 (1.14)	3.36 (1.48)	2.91 (1.41)
4. Change in mean FTE employment, balanced sample of stores ^c	– 2.28 (1.25)	0.47 (0.48)	2.75 (1.34)	1.21 (0.82)	0.71 (0.69)	– 2.16 (1.01)	3.36 (1.30)	2.87 (1.22)
5. Change in mean FTE employment, setting FTE at temporarily closed stores to 0 ^d	– 2.28 (1.25)	0.23 (0.49)	2.51 (1.35)	0.90 (0.87)	0.49 (0.69)	– 2.39 (1.02)	3.29 (1.34)	2.88 (1.23)

Notes: Standard errors are shown in parentheses. The sample consists of all stores with available data on employment. FTE (full-time-equivalent) employment counts each part-time worker as half a full-time worker. Employment at six closed stores is set to zero. Employment at four temporarily closed stores is treated as missing.

^aStores in New Jersey were classified by whether starting wage in wave 1 equals \$4.25 per hour ($N = 101$), is between \$4.26 and \$4.99 per hour ($N = 140$), or is \$5.00 per hour or higher ($N = 73$).

^bDifference in employment between low-wage (\$4.25 per hour) and high-wage ($\geq \$5.00$ per hour) stores; and difference in employment between midrange (\$4.26–\$4.99 per hour) and high-wage stores.

^cSubset of stores with available employment data in wave 1 and wave 2.

^dIn this row only, wave-2 employment at four temporarily closed stores is set to 0. Employment changes are based on the subset of stores with available employment data in wave 1 and wave 2.

For discussion: Explain columns (i)–(iii), including the DiD. Why do rows 3 and 4 differ? What is a full-time-equivalent employee?

Covariates in the Levels Model

Let X_{ik} be pre-treatment characteristics held fixed across the two periods.

$$Y_{it} = \alpha + \gamma T_i + \tau Post_t + \delta (T_i \times Post_t) + \sum_{k=1}^K \beta_k X_{ik} + \sum_{k=1}^K \theta_k (X_{ik} \times Post_t) + u_{it}.$$

- β_k allows baseline outcomes to differ with X_{ik} .
- θ_k allows the change over time to differ with X_{ik} .
- δ is the additional change associated with treatment, conditional on these characteristics.

From Levels to Differences with Covariates

For the same unit, subtract the pre-period equation from the post-period equation:

$$\Delta Y_i = \tau + \delta T_i + \sum_{k=1}^K \theta_k X_{ik} + \Delta u_i.$$

- α , γT_i , and $\sum_{k=1}^K \beta_k X_{ik}$ cancel.
- τ , δ , and θ_k retain exactly the same meanings.
- Card–Krueger Table 4 uses employment changes as the outcome, with chain and ownership indicators entered directly as controls.

Card and Krueger: Table 4

TABLE 4—REDUCED-FORM MODELS FOR CHANGE IN EMPLOYMENT

Independent variable	Model				
	(i)	(ii)	(iii)	(iv)	(v)
1. New Jersey dummy	2.33 (1.19)	2.30 (1.20)	—	—	—
2. Initial wage gap ^a	—	—	15.65 (6.08)	14.92 (6.21)	11.91 (7.39)
3. Controls for chain and ownership ^b	no	yes	no	yes	yes
4. Controls for region ^c	no	no	no	no	yes
5. Standard error of regression	8.79	8.78	8.76	8.76	8.75
6. Probability value for controls ^d	—	0.34	—	0.44	0.40

Notes: Standard errors are given in parentheses. The sample consists of 357 stores with available data on employment and starting wages in waves 1 and 2. The dependent variable in all models is change in FTE employment. The mean and standard deviation of the dependent variable are -0.237 and 8.825 , respectively. All models include an unrestricted constant (not reported).

^aProportional increase in starting wage necessary to raise starting wage to new minimum rate. For stores in Pennsylvania the wage gap is 0.

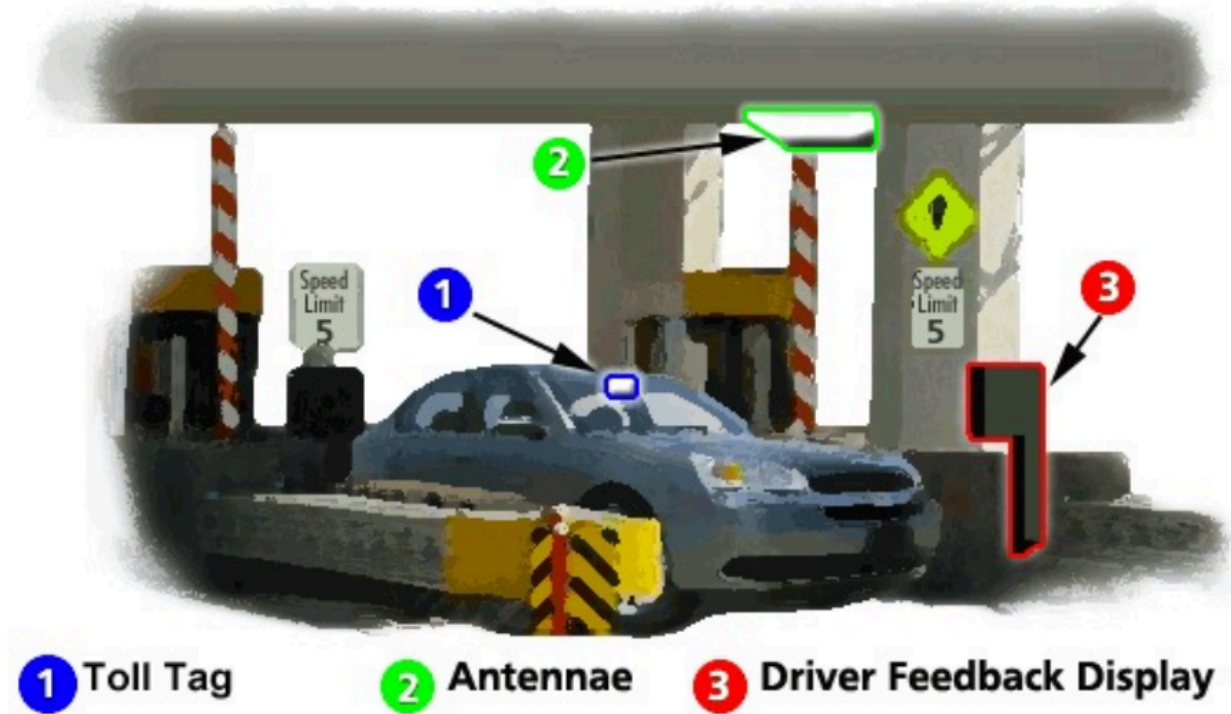
^bThree dummy variables for chain type and whether or not the store is company-owned are included.

^cDummy variables for two regions of New Jersey and two regions of eastern Pennsylvania are included.

^dProbability value of joint F test for exclusion of all control variables.

For discussion: Compare columns (i) and (ii). Why might chain and ownership matter? Separate the change in sample from the effect of adding controls.

Currie and Walker: The Policy



As your vehicle enters the toll lane, the toll tag (1) that is mounted on your vehicle's windshield is read by the antennae (2). As your vehicle passes through, your *E-ZPass*® account is charged the proper amount. Feedback is provided to you on an electronic display (3). If your vehicle does not have a toll tag, the system classifies you as a violator and cameras take photos of your license plate for processing.

E-ZPass allows electronic toll collection, reducing stops and queues at toll plazas. The study asks whether the resulting changes improve infant health nearby.

Currie and Walker: Data and Identification

For discussion: Define the treatment and control groups, the timing of treatment, and the birth outcomes. Why compare mothers near the same road network? State parallel trends for this design.

Where Are the Comparisons Made?

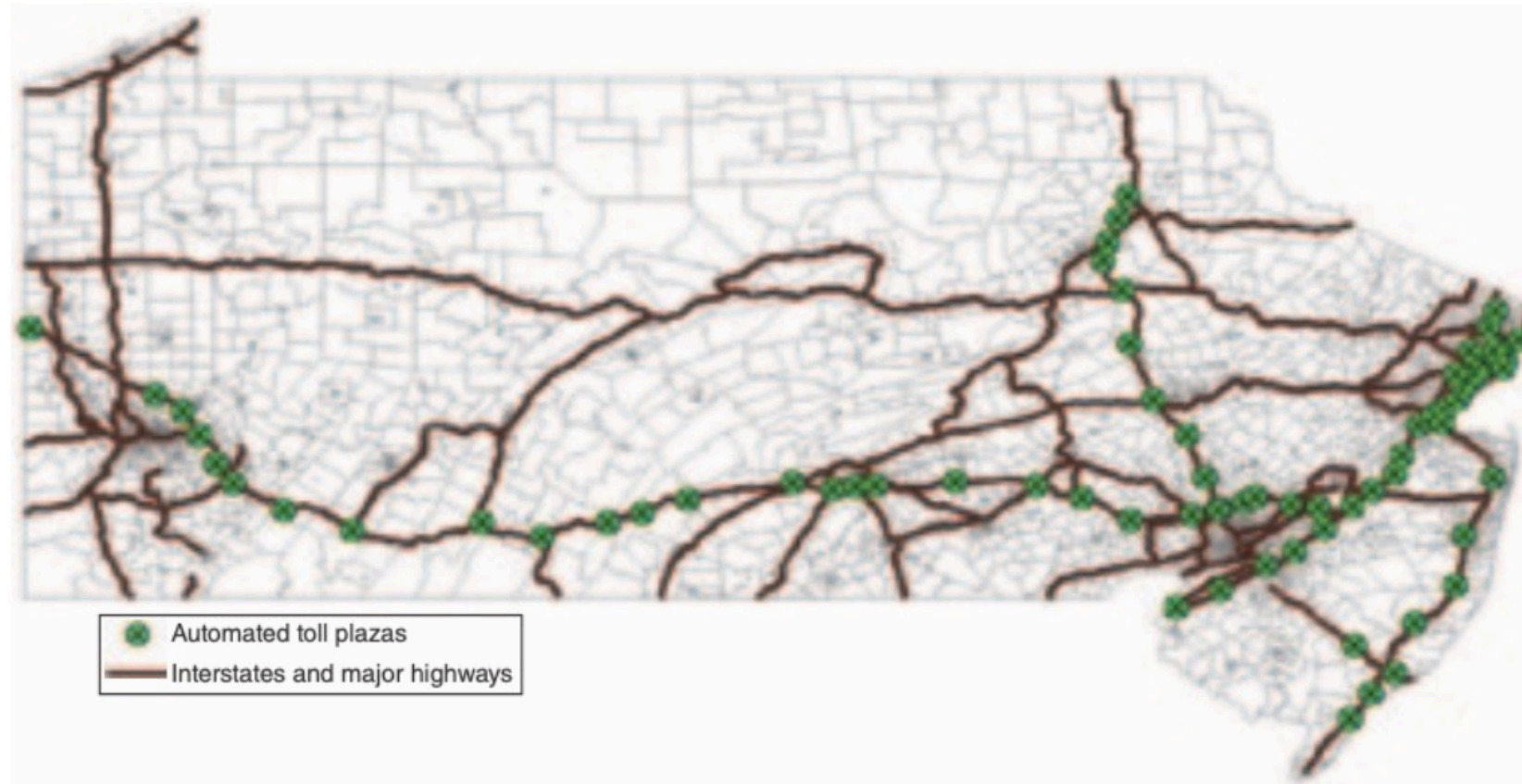


FIGURE 1. LOCATIONS OF TOLL PLAZAS AND MAJOR ROADWAYS IN NEW JERSEY AND PENNSYLVANIA

Currie and Walker (2011), Figure 1.

Currie and Walker: Table 3

TABLE 3—REGRESSIONS OF BIRTH OUTCOMES ON E-ZPASS ADOPTION
(Difference-in-Difference Specification)

	Prematurity (1)	Prematurity (2)	LBW (3)	LBW (4)
<i>Panel 1</i>				
<2km toll × after E-ZPass	−0.0085 [0.0039]**	−0.0086 [0.0034]**	−0.0094 [0.0032]**	−0.0093 [0.0028]**
<i>R</i> ²	0.0044	0.0034	0.0032	0.0028
<i>Panel 2</i>				
<1.5km toll × after E-ZPass	−0.0088 [0.0051]*	−0.0098 [0.0048]**	−0.0077 [0.0035]**	−0.0084 [0.0032]**
<i>R</i> ²	0.0042	0.0048	0.0035	0.0032
Maternal characteristics	No	Yes	No	Yes
Observations	405,802	405,802	409,673	409,673

Notes: Each coefficient is from a different regression. All regressions also included controls for being within 2 km (or 1.5 km) of a toll plaza, year of birth, month of birth, toll plaza indicators, an indicator for post E-ZPass, and distance to highway. Maternal characteristics include: mother black, mother Hispanic, mother education (<hs, hs, some college, college +), mother age (19–24, 25–34, 35+), smoking, multiple birth, gender, and birth order, and indicators for missing values. Standard errors are in brackets.

** Indicates that the estimate is statistically significant at the 95 percent level of confidence.

* Indicates significance at the 90 percent level of confidence.

For discussion: Interpret the prematurity and low-birth-weight estimates in Panel 1. Distinguish percentage points from percentages. How do the estimates change when maternal characteristics are included?

Currie and Walker: Table 2

TABLE 2—TESTING THE VALIDITY OF THE RESEARCH DESIGN: REGRESSIONS OF MATERNAL CHARACTERISTICS ON E-ZPASS ADOPTION
(Difference-in-Difference Specification)

	Black (1)	Hispanic (2)	Mother yrs. ed (3)	Dropout (4)	Teen mother (5)	Mother smoked (6)	Housing sale price (7)
<i>Panel 1</i>							
<2 km toll × after E-ZPass	−0.011 [0.011]	−0.01 [0.010]	0.037 [0.040]	−0.007 [0.005]	−0.001 [0.005]	0.005* [0.003]	0.149 [0.103]
Observations	397,201	406,641	406,198	397,201	412,884	402,590	252,343
<i>Panel 2</i>							
<1.5 km toll × after E-ZPass	−0.014 [0.055]	−0.01 [0.011]	0.013 [0.010]	−0.003 [0.006]	0.001 [0.003]	0.007** [0.003]	0.031 [0.106]
Observations	397,201	406,641	406,198	397,201	412,884	402,590	252,343

Notes: Each coefficient is from a separate regression. Each coefficient in columns 1–6 is from a regression that also included controls for being within 2 km (or 1.5 km) of a toll plaza, year of birth, month of birth, indicators for each toll plaza, an indicator for post E-ZPass at nearest toll plaza, and distance to highway. Housing sale price regressions in column 7 include year and month of sale, indicators for nearest toll plaza, an indicator for condo units, distance to highway, municipality fixed effects, square footage (in categories including dummies for missing), and age (in categories, including dummies for missing). Standard errors are in brackets.

** Indicates that the estimate is statistically significant at the 95 percent level of confidence.

* Indicates significance at the 90 percent level of confidence.

For discussion: Could changes in who lives near toll plazas explain the health results? What does this table tell us, and what uncertainty remains?

Currie and Walker: Table 7

TABLE 7—DIFFERENCE-IN-DIFFERENCES ESTIMATES OF EFFECTS OF E-ZPASS ON POLLUTION

	NO ₂ All control monitors (1)	SO ₂ All control monitors (2)	NO ₂ Random control 1 (3)	NO ₂ Random control 2 (4)	NO ₂ Random control 3 (5)	NO ₂ Random control 4 (6)	NO ₂ Random control 5 (7)
<2km toll × after E-ZPass	−0.108 [0.019]**	0.053 [0.034]	−0.208 [0.028]**	−0.090 [0.024]**	−0.065 [0.017]**	−0.181 [0.023]**	0.018 [0.038]
Observations	84,159	128,513	6,361	6,449	6,453	6,448	6,421

Notes: Each coefficient is from a separate regression. Columns 1 and 2 use all monitors over 2 km from a toll plaza as controls. Columns 3–7 each use a randomly selected control monitor. Regressions also included controls for being within 2 km of a toll plaza, year of birth, month of birth, indicators for each toll plaza, an indicator for post E-ZPass at nearest toll plaza, and distance to highway. Dependent variable is the log daily mean pollution level for the indicated pollutant. Standard errors are in brackets.

** Indicates that the estimate is statistically significant at the 95 percent level of confidence.

* Indicates significance at the 90 percent level of confidence.

For discussion: Compare the two pollutants. How does this evidence support the proposed mechanism? What does having only one monitor near a toll plaza imply?

What Can Threaten the Comparison?

- **Different underlying trends:** a local shock changes the treated group's outcome independently of the policy.
- **Anticipation:** behaviour changes before the designated treatment date.
- **Spillovers:** the policy affects the control group.
- **Composition:** the people or establishments observed change over time.

Controls and Evidence on Trends

Use institutional knowledge to motivate the comparison. Examine earlier trends when the data allow it.

Choose controls with a clear role in the design. Variables affected by the policy may remove part of the effect we want to estimate or introduce selection.

Uncertainty and the Level of Treatment

Many individuals can share the same policy and the same shocks.

- Restaurants share state-level economic conditions.
- Repeated observations on a restaurant can have correlated disturbances.
- Daily observations at a monitor can share persistent local conditions.

The number of observations is different from the number of independent policy comparisons.

Changes as the Dependent Variable

With the same units observed twice, subtract each unit's pre-period equation from its post-period equation:

$$\Delta Y_i = \tau + \delta T_i + \Delta u_i.$$

The intercept is the control group's average change. The coefficient on T_i is the difference between the groups' changes.

What a Credible DiD Design Needs

1. Clear policy timing and a well-defined outcome.
2. A plausible comparison group and an explicit parallel-trends argument.
3. Evidence on other changes, anticipation, spillovers, and composition.
4. Uncertainty appropriate to the available independent comparisons.

The control group supplies the **change** used to construct the treated group's counterfactual.

References

Card, D., and A. B. Krueger (1994). [Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania](#). *American Economic Review*, 84(4), 772–793.

Currie, J., and R. Walker (2011). [Traffic Congestion and Infant Health: Evidence from E-ZPass](#). *American Economic Journal: Applied Economics*, 3(1), 65–90.